CGI Number Facts Strategies Assessment

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I wish the fourth grade teachers would do more to teach number facts. My fifth graders don't know their multiplication facts and that makes it really hard for me to teach fractions and multi-digit multiplication.

I loved math until third grade when we started taking timed tests. I would study hard with my mom and I could do them all at the kitchen table, but my mind would go blank as soon as I got the test and the timer started.

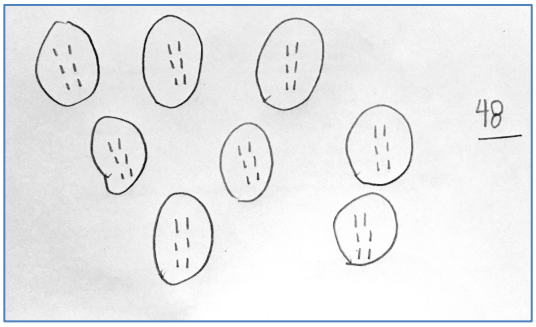
My third graders do great with our computer program that tests their number facts but when they are adding multi-digit numbers, I see them counting on their fingers.

My son cries every Thursday before school because that is the day of the timed tests.

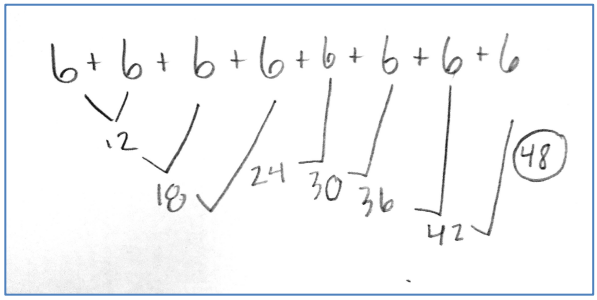
I hate how upset some of my students are when they have to take timed tests but I don't know how to assess my students' fluency without them.

Comments such as these have been commonplace in teachers' lounges since I began teaching over 25 years ago. Many teachers are distressed and confused when it comes to teaching number facts.

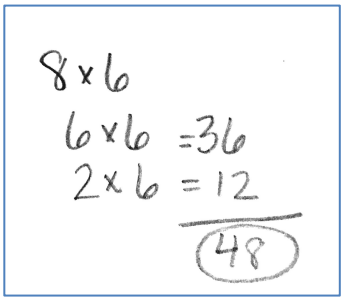
Fluency with number facts is very important to CGI teachers. In fact, fluency with number facts may be more important in CGI classrooms than in traditional classrooms. When children in CGI classes first encounter number facts, they are encouraged to solve them in whatever way makes sense to them. For example, here is how Maria, a first grader, solved the problem; Marcia has 8 bags with 6 candies in each bag. How many candies does Marcia have all together?



As children's understanding of multiplication grows, they might solve the problem 8×6=n as Tony, a third grader did:



As children continue to mature, they will start to use properties of multiplication along with facts they already know to figure out facts they don't know. For example, Annette, a fourth grader solved 8×6= n as follows:



Number fact instruction is essential in CGI classrooms because it provides opportunities for students to develop an understanding of how operations work and engage with concepts about operations. Maria's strategy developed an understanding of multiplication as grouping. Tony's strategy developed an understanding of the relationship between multiplication and addition. Annette's strategy, which could be represented with the equation,

$$8\times6 = (6+2)\times6 = (6\times6) + (2\times6),$$

used the Distributive Property of Multiplication over Addition. These children engaged with all of these concepts while solving the number fact 8 times 6.

Students in CGI classes learn number facts by solving problems using their own strategies and discussing their classmates' strategies with the support of a teacher who helps them focus on the concepts embedded in each strategy. Not only does this approach support students in learning mathematical concepts, it also supports students' fluency with number facts. When students learn facts through memorization, each fact is an isolated piece of information. Memorizing 8×6 = 48 isn't much different than memorizing that the symbol for Gold is Au. Isolated pieces of information are difficult to remember. (Did you remember that the symbol for Gold is Au? You probably memorized this fact in High School.) We have all witnessed phenomena such as the student who passed the timed test for multiplication at the end of fourth grade and wasn't able to pass the same test at the beginning of fifth grade. Research shows that information learned through memorization is especially difficult to access during times of

stress. This research explains why some children do fine with flashcards at the kitchen table only to fail miserably on the timed test in school the next day. Although it may seem to take longer to teach facts with the CGI approach, when we consider how often we need to reteach number facts learned through memorization, we see that the CGI approach is actually more efficient than memorization.

When students learn number facts by discussing different strategies with the support of a teacher who helps them focus on concepts, number facts are connected to other mathematical concepts. Information connected to other concepts is easier to remember, stored in a manner that is accessible during times of stress, and can be recreated if forgotten. (Annette will eventually just remember that 8 times 6 is 48 but if she ever forgets, she can go back and figure it out using 6×6 plus 2×6.) Most students who learn number facts using this approach will eventually know most facts from memory. Knowing a number fact from memory doesn't mean the fact was learned by memorizing. Teachers shouldn't worry if a student has a few facts that they need to 3-5 seconds to figure out; taking a few seconds to figure out a few facts will not affect students' success in later mathematics.

The information obtained from this assessment helps teachers support students to learn mathematical concepts in conjunction with developing fact fluency. Such information is different from the information obtained from a traditional paper and pencil test or computer number fact game. For example, a test for subtraction facts may tell you that a student got correct answers for all of the minus 0, 1, 2, and 3 facts but got only 50% of the other subtraction facts correct in the allotted time. One student may obtain this result and be Directly Modeling for many subtraction facts and simply not have time to complete the test. Another student may obtain this same result and be using Relational Thinking on several subtraction problems. If Relational Thinking is new to this student, each strategy may take some processing time and the student may not have time to complete the test. A third student may obtain this result and be using Counting Strategies and might be struggling when counting backwards and getting several problems wrong. Each of these students needs different instruction to build their fact fluency and develop an understanding of how subtraction works.

Teaching with understanding requires teachers to build upon what students already understand. This assessment enables you to know what your students understand about a given set of number facts so you can design instruction to increase their fluency with number facts while also developing their understanding of the properties of operations. Specifically, it will help you assess:

1. What strategies a student uses on a set of number facts – Direct Modeling, Counting, Relational Thinking or Recall.
2. For students using Relational Thinking, what Relational Thinking strategies they use and whether or not these strategies are efficient.

This assessment is designed to be one of many methods of collecting information about students' understanding of number facts. It was written for students in CGI classrooms with the assumption that these students will solve many different types of math problems – word problems being the most typical types of problems they solve. Students will use their own

strategies to solve problems and share their thinking with their teachers and classmates. Teachers gain a great deal of information about their students from listening to and observing students solving problems. Some of your students may not be ready for this assessment because they need a story in order to reason about operations. Closely monitor these students during class work to determine when they are ready for this assessment.

Administering the CGI Number Facts Strategies Assessment

The goal of this assessment is for you to gain information about students' number fact strategies so that can you design instruction to build fact fluency. We recommend that you follow these guidelines until you feel comfortable with administering the assessment. ***Once you feel comfortable with the assessment, adapt these directions if another course of action would give you better information about your students' thinking.***

1. Meet with one student at a time.

2. Do not provide students with paper, pencil, or manipulatives. Provide these tools during instruction, but don't provide them during this assessment. As you gain experience with this assessment, there may be times that you will provide some tools for some problems. You can usually assess what students do with tools during instruction so you don't need to provide tools now.

3. Have the students put their hands where you can see them so you can tell if they are using them to solve the problem.

4. Place the student copy of the assessment in front of the student. Ask the student one fact at a time. You can read the fact to the student or just ask the student to solve the next one on the list. Cover the problems the student hasn't solved with a piece of thick paper. Ask the student to move the paper down the student sheet one fact at a time so the student can see facts he or she has already solved but not facts he or she is going to solve. Neither you nor the student will write anything on the student copy[1].

5. After the student provides an answer, ask the student how s/he got that answer if you don't already know how the student got the answer. If you saw the student count on fingers, you don't need to ask (but you could if you wanted to). If the student answers immediately, you don't need to ask; you could assume for now that the student recalled the fact from memory. If the student answers many facts in 1-2 seconds you will further assess their strategies at the end of the interview (see #9). You should always feel that you could ask the student to explain a strategy if you want to.

6. If a student is struggling a great deal, you might move to the Entry Level Assessment for the same operation. Once you gain experience with using this assessment, you will have a better sense of when to continue on with a student and when to stop the assessment. For now, if a student isn't frustrated, you might want to finish the assessment to see what information you may gain from the remaining problems.

7. Write down how the student solved the problem. If the student Directly Modeled, you can write DM. If the student used a Counting Strategy, you can write C and then the numbers that student said (so that you could later assess what counting strategy they

[1] If you have a student with a language or communication difficulty and the assessment would be more accurate if s/he wrote the answers, certainly allow the student to write answers.

used). If the student used Relational Thinking you can write RT and then write what the student says. Here are some examples of what you might write:

Number fact	**If student says**	**You could write down**
3 + 8	8, 9, 10, 11, 11	C 8, 9, 10, 11
8 + 4	"8 and 2 is 10 and 2 more is 12"	RT 8 + 2 → 10 + 2 → 12
14 - 6	"14 minus 4 is 10 minus 2 more is 8"	RT 14 – 4 →10 – 2 → 8
3 × 6	"I knew that 6 and 6 was 12 and then 13, 14, 15, 16, 17, 18"	RT 6 + 6 → 12, 13, 14, 15, 16, 17, 18
8 × 6	"I did 8 times 5 is 40 and then I knew I needed one more 8 so 48"	RT 8 × 5 → 40 + 8 → 48

8. You may wish to put a line over the answer if the student gets the answer wrong. This way, if the student sees your paper, they don't know if their answers are right or wrong. When you look back at the interview, you could then quickly see which problems the student got wrong.

9. At the close of the interview, if the student answered several facts within 2 seconds, go back and question to see if the student can use relational thinking. Ask specifically about some of the problems in **bold.** If the student repeatedly says, "I have that one memorized," try asking something like, "If you didn't have it memorized, how could you figure it out?" OR "If someone disagreed with you, how could you prove your answer is right?"

10. At the close of the interview, go back and ask the student to explain her/his thinking for any facts you might be curious about. Some teachers ask about facts that the student got wrong at this time.

11. Complete the summary sheet soon after the interview. Each student will have a separate summary sheet where you can record information from 3 different assessment dates.

12. To assess efficient rate, we recommend these guidelines:
    1st grade: 4 seconds per fact
    2nd and 3rd grade: 3 seconds per addition fact; 4 second per subtraction or multiplication fact
    4th - 6th grade: 3 seconds per fact
    7th grade and higher: 2 second per fact
13. Although there is a separate assessment for each operation, you should not wait until a child is fluent with one operation before assessing their strategies on other operations. Once a student begins using relational thinking for addition, you should start assessing the student's strategies for subtraction and multiplication. Once a student begins using relational thinking for multiplication, you should start assessing their strategies for

division. With experience you will be able to integrate these assessments into one experience based on the student's understanding of a particular operation.

Please remember that the goal is to solve MOST facts at an efficient rate; ***having a few facts that take longer than this rate will not affect students' success in mathematics.***

If you are working with a student who doesn't seem to be working at an efficient rate you may wish to note how long the student takes to solve each problem. Some teachers put a dot next to the number fact for each 3 seconds the student takes to work on that fact. You can then quickly see which problems the student is taking a long time on. Ensure you allow the student to take the time s/he needs to solve each fact but don't feel you need to ask all the facts on the assessment if the student is working slowly.

Name_____ Date _____

Addition Facts Assessment – teacher note taking sheet CGI NFS Assessment

Fact	Answer	Strategy
5 + 5		
7 + 3		
8 + 3		
2 + 7		
6 + 4		
5 + 6		
4 + 8		
2 + 8		
6 + 6		
7 + 7		
7 + 5		
7 + 6		
9 + 7		
5 + 7		
6 + 8		
8 + 9		
6 + 9		
8 + 7		

Addition Facts -- Student Sheet

5 + 5
7 + 3
8 + 3
2 + 7
6 + 4
5 + 6
4 + 8
2 + 8
6 + 6
7 + 7
7 + 5
7 + 6
9 + 7
5 + 7
6 + 8
8 + 9
6 + 9
8 + 7

Name_____ Date _____

ENTRY LEVEL ADDITION FACTS Assessment – teacher note taking sheet

Fact	Answer	Strategy
0 + 8		
1 + 2		
1 + 4		
2 + 2		
2 + 3		
3 + 2		
4 + 4		
4 + 5		
3 + 3		
3 + 4		
5 + 5		
5 + 4		
4 + 6		
3 + 7		
2 + 4		
2 + 5		

Addition Facts 2 -- Student Sheet

0 + 8
1 + 2
1 + 4
2 + 2
2 + 3
3 + 2
4 + 4
4 + 5
3 + 3
3 + 4
5 + 5
5 + 4
4 + 6
3 + 7
2 + 4
2 + 5

Name_____ Date _____

Subtraction Facts Assessment – teacher note taking sheet CGI NFS Assessment

Fact	Answer	Strategy
4 – 2		
10 – 5		
6 – 2		
8 – 4		
12 – 4		
14 – 7		
11 – 2		
12 – 8		
14 – 6		
15 – 7		
13 – 6		
12 – 5		
17 – 8		
12 – 3		
16 – 9		
18 - 9		
15 - 8		
13 - 9		

Subtraction Facts -- Student Sheet

4 – 2
10 – 5
6 – 2
8 – 4
12 – 4
14 – 7
11 – 2
12 – 8
14 – 6
15 – 7
13 – 6
12 – 5
17 – 8
12 – 3
16 – 9
18 – 9
15 – 8
13 – 9

Name_____ Date _____

ENTRY LEVEL Subtraction FACTS Assessment – teacher note taking sheet

Fact	**Answer**	**Strategy**
3 + 3		
6 – 3		
6 + 4		
6 – 0		
10 – 4		
10 – 3		
11 – 1		
10 – 2		
11 – 3		
12 – 4		
6 + 6		
12 – 6		

Subtraction Facts 2 -- Student Sheet

3 + 3
6 – 3
6 + 4
6 – 0
10 – 4
10 – 3
11 – 1
10 – 2
11 – 3
12 – 4
6 + 6
12 – 6

Name_____ Date _____

Multiplication Facts Assessment – teacher note taking sheet

Fact	**Answer**	**Strategy**
3×5		
7×2		
6×5		
4×4		
3×6		
8×5		
5×6		
4×3		
4×9		
6×6		
5×7		
6×8		
7×6		
9×8		
8×6		
7×8		
4×8		
6×9		

Multiplication Facts -- Student Sheet

3×5
7×2
6×5
4×4
3×6
8×5
5×6
4×3
4×9
6×6
5×7
6×8
7×6
9×8
8×6
7×8
4×8
6×9

Name_____ Date _____

ENTRY LEVEL Multiplication Facts Assessment – teacher note taking sheet

Fact	**Answer**	**Strategy**
2×6		
6×2		
7×1		
5×2		
2×5		
3×5		
1×8		
3×3		
4×3		
3×4		
4×4		
5×5		

Multiplication Facts 2 -- Student Sheet

2×6
6×2
7×1
5×2
2×5
3×5
1×8
3×3
4×3
3×4
4×4
5×5

Name_____ Date _____

Division Facts Assessment – teacher note taking sheet

Fact	Answer	Strategy
9 ÷ 9		
10 ÷ 2		
12 ÷ 4		
20 ÷ 5		
36 ÷ 6		
45 ÷ 5		
32 ÷ 4		
21 ÷ 3		
24 ÷ 6		
42 ÷ **6**		
56 ÷ 7		
72 ÷ 8		
48 ÷ 8		
54 ÷ 9		
63 ÷ 7		
63 ÷ 9		
54 ÷ 6		

Division Facts -- Student Sheet

9 ÷ 9
10 ÷ 2
12 ÷ 4
20 ÷ 5
36 ÷ 6
45 ÷ 5
32 ÷ 4
21 ÷ 3
24 ÷ 6
42 ÷6
56 ÷ 7
72 ÷ 8
48 ÷ 8
54 ÷ 9
63 ÷ 7
63 ÷ 9
54 ÷ 6

Name of Student_____

Addition Fact Assessment Summary Sheet CGI NFS Assessment

Date			
Did student use Relational Thinking on any problems or have any facts at recall level?			
Strategy (or strategies) used most often (Direct Modeling, Counting, Relational Thinking, Recall)			
Relational Thinking - Evidence of use of Commutative Property of Addition (Look at: 2 + 7; 2 + 8 and others.)			
Relational Thinking - Evidence of being able to build up in chunks to find total (i,.e. uses the **associative property of addition**)Write out relationship used.			
Uses appropriate (efficient) strategies for given facts (always, sometimes, seldom)			

Name of Student_____

Subtraction Fact Assessment Summary Sheet CGI NFS Assessment

Date			
Did student use Relational Thinking on any problems or have any facts at recall level?			
Strategy (or strategies) used most often(Direct Modeling, Counting, Relational Thinking, Recall)			
Evidence of use of relationship between addition and subtraction			
Evidence of being able to build up in chunks to find the difference (e.g. for 12 – 8, 8 + 2 → 10 + 2 → 12, so 12 – 8 = 4)			
Evidence of being able to take away in chunks to find difference (e.g. for 12 – 4, 12 – 2 → 10 – 2 → 8 so 12 – 4 = 8)			
Uses appropriate (efficient) strategies for given facts (always, sometimes, seldom)			

Name of Student_____

Multiplication Fact Assessment Summary Sheet CGI NFS Assessment

Date			
Did student use Relational Thinking on any problems or have any facts at recall level?			
Strategy (or strategies) used most often (Direct Modeling, Counting, Relational Thinking, Recall)			
Evidence of use of Commutative Property of Multiplication (Look at: 7×2; 5×n; 8×6)			
Evidence of use of Distributive property of Multiplication over Addition. (Write out relationship used.)			
Evidence of use of associative property of multiplication (Write out relationship used.)			
Uses appropriate (efficient) strategies for given facts (always, sometimes, seldom)			

Name of Student_____

Division Fact Assessment Summary Sheet CGI NFS Assessment

Date			
Did student use Relational Thinking on any problems or have any facts at recall level?			
Strategy (or strategies) used most often (Direct Modeling, Counting, Relational Thinking, Recall)			
Evidence of use of relationship between multiplication and division			
Evidence of use of other use of relational thinking (What properties were used?) (Write out relationship used.)			
Uses appropriate (efficient) strategies for given facts (always, sometimes, seldom)			